WJEC (Wales) Physics A-level

Topic 1.3: Dynamics

Notes

Newton's 3rd Law

Force

A force is a **push or pull** which causes the motion of a body. There are multiple types of forces. Forces are vector quantities (so they have a magnitude and direction) and their magnitude is usually measured in Newton's (*N*).

Interactions

Newton's 3rd law states that if a body A exerts a force of magnitude F on a body B, the body B will exert a force of **equal magnitude** but **opposite direction** (-F) on body A.

It is sometimes stated as 'for every action there is an equal and opposite reaction'.

Newton's 3rd law is used to explain many situations, **rocket propulsion** as an example. When the rocket accelerates the hot exhaust gases out the end a reaction force is produced acting on the rocket – accelerating it upwards.

Newton's 2nd Law

Free body diagrams

These are a type of diagram where you draw just the body in question and the forces acting on it. For example, a book on a table.

Reaction force from table

Weight of book
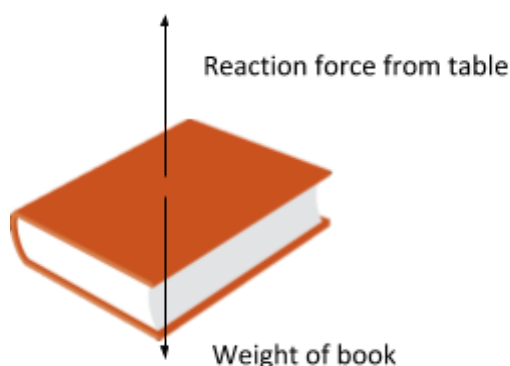
Scale diagrams where the arrows are shown to be longer for larger magnitude forces can be used if force magnitudes are not given.

The Law

Newton's second law is often applied after referencing a free body diagram. Put simply the second law states that $\sum F = ma$. Equivalently: *The resultant force on a body in a **direction** is equal to the mass of the body multiplied by the acceleration of the body **in that direction***.

The key thing to remember is that both force and acceleration are **vector quantities**. This form of Newton's second law only applies when the mass of the body is constant.

We will now look at a few examples.
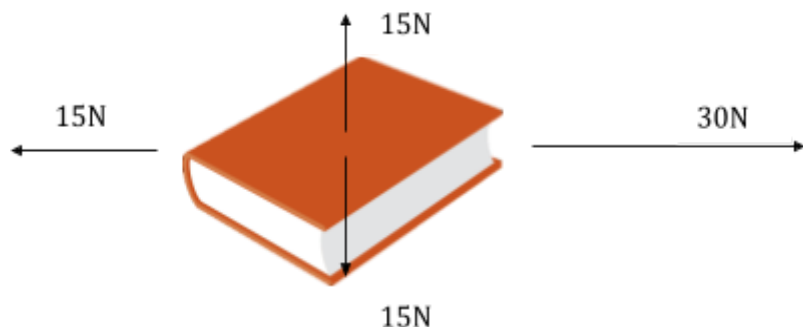
In this case we will look at the horizontal and vertical motion separately. Vertically, the resultant force is $0N$ and therefore the book will remain in vertical equilibrium.

Horizontally, there is a resultant force of $30N - 15N = 15N$ to the right. The weight of the book is $10N$ and so using the approximation that $g = 10 \ N/kg$ we have the mass of the book to be $1 \ kg$.
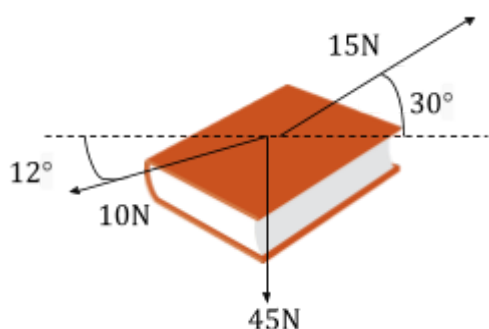
Now, using Newton's second law,

$$15 = ma = 1 \times a$$

$$\Rightarrow a = 15 \ m/s^2$$

The acceleration is to the right because the resultant force was to the right.

The next example is a bit trickier.

The weight of the body is $45N$ and so its mass is $4.5kg$.

It may be useful to bring in some new notation shown by applying it to the above example.

$$Nl2 \ on \ book \rightarrow$$

$$15cos\,(30) - 10cos\,(12) = 4.5 \times a$$

$$\Rightarrow 4.5a = 3.2089\ldots N$$

$$\Rightarrow a = 0.713\ldots = 0.713 \ m/s^2 \ (3sf)$$

$$Nl2 \ on \ book \downarrow$$

$$45 + 10sin(12) - 15sin(30) = 4.5 \times a$$

$$\Rightarrow 4.5a = 39.57\ldots N$$

$$\Rightarrow a = 8.7953\ldots = 8.80 \ m/s^2 \ (3sf)$$

The 'NL2' means to apply Newton's second law. Then you specify **which body to apply it on** (the reason behind this is so you do not get confused between bodies in large systems). Then you specify which direction you apply it in. This is so that you find the **resultant force in the right direction** and the acceleration in that same direction.

So, we found that the book will accelerate at $0.713 \ m/s^2$ to the right and $8.80 \ m/s^2$ downwards. The magnitude of the resultant acceleration can be found by applying the **Pythagorean theorem**.
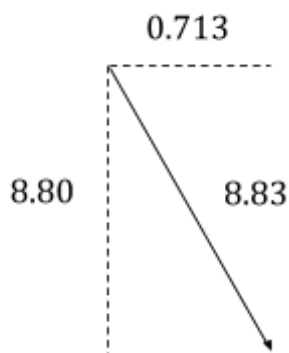
This gives the magnitude of the acceleration as $\sqrt{\left(8.8^2 + 0.713^2\right)} = 8.83 \ m/s^2$

Momentum and Force

Linear momentum is given by the following equation

$$p = mv$$

p is **linear momentum**, m is mass and v is velocity. The units are usually *kg* for mass and *ms⁻¹* for velocity and therefore the units of linear momentum are *kgms⁻¹*. **Momentum is a vector quantity and is in the direction of the velocity**.

Force is related to momentum in that **force is the rate of change of momentum**.

An example:

The block above changes velocity from 2 ms⁻¹ to the right to 7 ms⁻¹ to the right. The change takes 2 seconds. Calculate the average force on the block.

$$F = \frac{\Delta p}{\Delta t} = \frac{mv - mu}{\Delta t} = \frac{3(7) - 3(2)}{2} = \frac{15}{2} = 7.5 \; N$$

Conservation of Momentum

In closed systems (where no external forces are applied) the **total momentum of the system does not change**. It is conserved. Momentum may be transferred between bodies, but the total momentum must remain constant.

Collisions

We can use the conservation of momentum to find out information about bodies in collisions.

An example:

The diagram shows two particles heading towards each other. Given that the 5kg particle moves to the right with a speed of 1 m/s after the collision, find the final velocity of the 2kg particle.
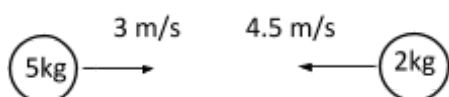
Total momentum before collision to the right $= 5\,(3) - 2\,(4.5) = 6 kgm/s$

Let the speed of the 2kg particle after the collision be v m/s.

Total momentum after collision to the right $= 5\,(1) + 2\,(v) = 5 + 2v \; kgm/s$

As total momentum must be conserved we can equate the two expressions and we have

$$6 = 5 + 2v$$

$$2v = 1$$

$$v = 0.5 \; m/s$$

Therefore, the velocity of the 2kg particle is 0.5 m/s to the right.

Inelastic and Elastic Collisions

An **inelastic** collision is one in which **kinetic energy is lost**.

An **elastic** collision is one in which **no kinetic energy is lost**.
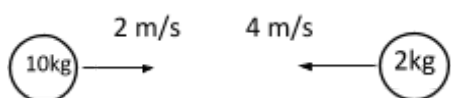
Using the above example:

Total kinetic energy before the collision $= \frac{1}{2}(5)(3)^2 + \frac{1}{2}(2)(4.5)^2 = 22.5 + 20.25 = 42.75J$

Total kinetic energy after the collision $= \frac{1}{2}(5)(1)^2 + \frac{1}{2}(2)(0.5)^2 = 2.5 + 0.25 = 2.75J$

Therefore, as kinetic energy is lost, this is an inelastic collision.

Now, we will look at an elastic collision:

A 10kg particle is moving towards a 2kg particle at 2m/s. The 2kg particle is moving at 4m/s. After the collision, the 10kg particle is stationary, find the speed of the 2kg particle.

Total momentum before to the right $= 10(2) - 2(4) = 12 \ kgm/s$

Total momentum after to the right $= 2u \ kgm/s$

Hence,

$$12 = 2u$$

$$u = 6 \ m/s$$

In this case we had an elastic collision:

Total kinetic energy before collision $= \frac{1}{2}(10)(2)^2 + \frac{1}{2}(2)(4)^2 = 20 + 16 = 36J$

Total kinetic energy after collision $= \frac{1}{2}(10)(0)^2 + \frac{1}{2}(2)(6)^2 = 0 + 36 = 36J$

Total kinetic energy before collision = total kinetic energy after the collision

In our last example, we will explain how to find the speeds of both particles in an elastic collision when not given any of them.

Find the speeds of both particles after they undergo an elastic collision.
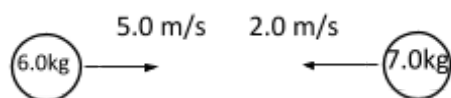
We will draw a diagram for after the collision

We cannot be certain (without calculation) of the directions of motion of the particles after, but if we find one of the speeds to be negative then it means it is in the opposite direction.

Total momentum before the collision to the right $= 6(5) - 7(2) = 30 - 14 = 16 \, kgm/s$

Total momentum after the collision to the right $= 6v + 7u \, kgm/s$

So, by the conservation of momentum $16 = 6v + 7u$

$$u = \frac{16 - 6v}{7}$$

Then, because we know the collision is elastic we can equate the kinetic energy before and after:

$$\tfrac{1}{2}(6)(5)^2 + \tfrac{1}{2}(7)(2)^2 = \tfrac{1}{2}(6)(v)^2 + \tfrac{1}{2}(7)(u)^2$$

$$75 + 14 = 3v^2 + \tfrac{7}{2}u^2$$

$$178 = 6v^2 + 7u^2$$

$$178 = 6v^2 + 7\left(\tfrac{16-6v}{7}\right)^2$$

$$1246 = 42v^2 + (16 - 6v)^2$$

$$1246 = 42v^2 + (256 - 192v + 36v^2)$$

$$78v^2 - 192v - 990 = 0$$

Using the quadratic formula, we have,

$$v = \frac{-(-192) \pm \sqrt{(-192)^2 - 4(78)(-990)}}{2(78)}$$

$$v = 5 \, or -\tfrac{33}{13}$$

Now, the corresponding values of u are,

$$u = -2 \, or \, \tfrac{58}{13}$$

If $v = 5 \, and \, u = -2$ then this means that after the collision the particles are still moving towards each other. This does not make sense. If $v = -\tfrac{33}{13} \, and \, u = \tfrac{58}{13}$ then the particles are moving away from each other and the situation **makes sense**.

We should give our answers to an appropriate number of significant figures. In this case, we will give them to two significant figures as the data provided is to two.

So, the speed of the 6.0kg particle is 2.5 m/s (2sf) and the speed of the 7.0kg particle is 4.5 m/s (2sf).

If you are solving quadratics like these, you will often get two real solutions and you should check both to see which one will **work in the physical situation**.